विद्या सर्वार्थि साधिका

ANANDALAYA

PRE – BOARD EXAMINATION

Class: XII

Subject : Mathematics (041) M.M.: 80

Date : 20 – 12 – 2023 Time : 3 Hours

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

SECTION A

1. Let A = {a, b, c}, then the total number of distinct relations in set A are _____ . (1)

   (A) 64 (B) 32 (C) 256 (D) 512

2. If $y = \log x, then \frac{d^2y}{dx^2} =$ _____. (1)

   (A) $-\frac{1}{x^2}$ (B) $\frac{1}{x}$ (C) 1 (D) x

3. The function $f(x) = 4 - 3x + 3x^2 - x^3, x \in R$ is _____. (1)

   (A) decreasing function (B) increasing function

   (C) strictly increasing on R (D) neither increasing not decreasing on R

4. What is the domain of the function $\cos^{-1}(2x - 3)$? (1)

   (A) [– 1, 1] (B) (1, 2) (C) (– 1, 1) (D) [1, 2]

5. The value of p for which vectors $2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\hat{\imath} - p\hat{\jmath} + 4\hat{k}$ are orthogonal is _____. (1)

   (A) 12 (B) – 12 (C) 14 (D) – 14

6. Area bounded by the curve y = sin x and the x-axis between x = 0 and x = 2π is _____, (1)

   (A) 2 sq units (B) 0 sq units (C) 3 sq units (D) 4 sq units

7. Given function $f(x) = \frac{x^2-4}{x-2}$ and $g(x) = x + 2, x \in R$. Then which of the following is correct? (1)

   (A) f is continuous at x = 2, g is continuous at x = 2.

   (B) f is continuous at x = 2, g is not continuous at x = 2.

   (C) f is not continuous at x = 2, g is continuous at x = 2.

   (D) f is not continuous at x = 2, g is not continuous at x = 2.

8. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then which one is true from the following? (1)

   (A) $\vec{a}$ is parallel to $\vec{b}$ (B) $\vec{a}$ is perpendicular to $\vec{b}$ (C) $|\vec{a}| = |\vec{b}|$ (D) $\vec{a} = \vec{b}$

9. The corner points of the feasible region for a Linear Programming problem are P (0,5), Q (1, 5), R (4, 2) and S (12,0). The minimum value of the Objective function Z = 2x + 5y is at the ____ point. (1)

   (A) P (B) Q (C) R (D) S

10. Principal branch of $\tan^{-1} x$ is _____. (1)

    (A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left(0, \frac{\pi}{2}\right)$ (C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

11. If A and B are independent events such that P (B/A) = 2/5, then find $P(B')$. (1)

(A) 3/5 (B) 2/5 (C) 1/5 (D) 4/5

12. The objective function for a given linear programming problem is $Z = ax + by - 5$. If Z attains same value at (1,2) and (3, 1), then which one is true from the following? (1)

(A) a + 2b = 0 (B) a + b = 0 (C) a = b (D) 2a – b = 0

13. Which one is the direction ratio of the given line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$? (1)

(A) 2, 6, 3 (B) – 2, 6, 3 (C) 2, – 6, 3 (D) none of these.

14. The diagonal elements of a skew symmetric matrix are _____. (1)

(A) all zeros (B) are all equal to some scalar k (≠0)

(C) can be any number (D) all are one only.

15. $\int \frac{x^2}{1+x^3} dx =$ _____. (1)

(A) $\frac{2}{3x} + C$ (B) $2\log x + C$ (C) $\frac{1}{3}\log|1 + x^2| + C$ (D) $3\log|1 + x^2| + C$

16. If the matrix $m \times n$, and for matrix B, AB and BA both are defined, then _____is the order of matrix B. (1)

(A) $m \times n$ (B) $n \times n$ (C) $m \times m$ (D) $n \times m$

17. If p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then find the value of 2p – 3q. (1)

(A) 7 (B) – 7 (C) 3 (D) – 3

18. If $A = \begin{bmatrix} k & 10 \\ 7 & k-3 \end{bmatrix}$ is a singular matrix, then value(s) of k is/are _____. (1)

(A) – 10 , 7 (B) 10, – 7 (C) 10 (D) 7

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

19. Assertions (A) : Two coins are tossed simultaneously. The probability of getting two heads if it is known that at least one head comes up is 1/3. (1)

Reason (R) : Let E and F be two events with a random experiment then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

20. Assertions (A) : if $\int_0^a f(x)dx = 5$ then $\int_0^a f(a-x)dx = 5$ (1)

Reason (R) : $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

SECTION B

21. Find the cartesian equation of the line which passes through the point (– 2, 4, – 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. (2)

22. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$. (2)

OR

Find x and y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

23. Three persons A, B and C fire a target in turn. Their probabilities of hitting the target are 0.2, 0.3 and 0.5 respectively, Find the probability that target is hit by any of A, B or C. (2)

24. Find the vector of magnitude 6, perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$. (2)

OR

If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$ *and* $|\vec{c}|=7$, then find the angle between $\vec{a}$ *and* $\vec{b}$.

25. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$. (2)

SECTION C

26. Given $f(x)=\sin x$, check if function f is one-one for $(i)(0,\pi)$ $(ii)\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. (3)

27. Find the area of the region bounded by the curve $y=x^2$ and the line y = 4. (3)

28. Evaluate: $\int_0^e \frac{dx}{x\sqrt{1-(\log x)^2}}$. (3)

OR

Evaluate: $\int \frac{1-\cos x}{1+\cos x}\,dx$.

29. Show that $y=ae^{3x}+be^{-x}$ is a solution of $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$. (3)

30. Find the value of p, so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{5z-10}{11}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other. (3)

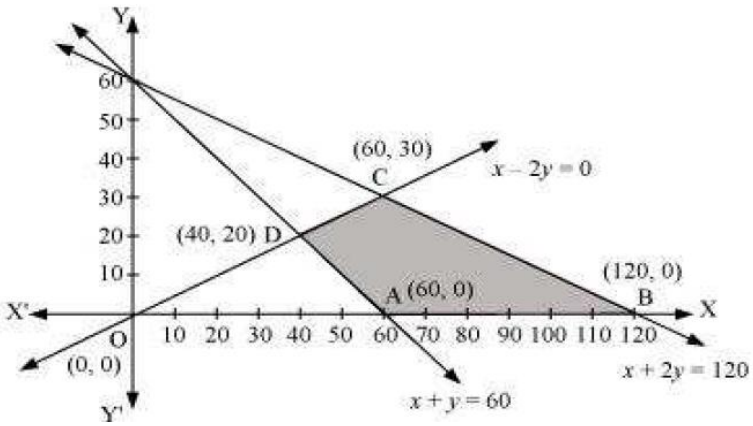
31. Solve given LPP graphically: (3)
Objective: Maximize $Z=-x+2y$,
Subject to the constraints: $x\geq 3, x+y\geq 5, x+2y\geq 6, y\geq 0$.

OR

Observe the graph of the following LPP for Minimise Z = 5x + 10y, where the subject to constraints

$x+2y\leq 120$,
$x+y\geq 60$,
$x-2y\geq 0$
and $x,y\geq 0$.

Find the point on which Z is minimum also find the minimum value.



SECTION D

32. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$. (5)

33. Find the particular solution of the differential equation $(x+y)dy+(x-y)dx=0$, given that when x = 1, y =1. (5)

OR

Solve the differential equation $\frac{dy}{dx}=-\left[\frac{x+y\cos x}{1+\sin x}\right]$.

34. If $x=a\sin 2t(1+\cos 2t)$ and $y=b\cos 2t\ (1-\cos 2t)$, show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}}=\frac{b}{a}$. (5)

35. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ $find\ (AB)^{-1}$ (5)

OR

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, $find\ A^{-1}$. Hence solve the system of equations using matrix method.

$2x + y - 3z = 13, \quad 3x + 2y + z = 4, \quad x + 2y - z = 8.$

SECTION –E

This section comprises of 3 case- study/ passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.

36. Suman was doing a project on a school survey, on the average number of hours spent on study by students selected at random. At the end of survey, Suman prepared the following report related to the data. Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.2, & if\ x = 0 \\ kx, & if\ x = 1\ or\ 2 \\ k(6 - x), & if\ x = 3\ or\ 4 \\ 0, & otherwise \end{cases}$$

Based on the above information, answer the following questions.

(i) Find the value of k. (1)

(ii) What is the probability that the average study time of students is not more than 1 hour? (1)

(iii) What is the probability that the average study time of students is at least 3 hours? (2)

OR

(iii) What is the probability that the average study time of students is at least 1 hour?

37. Muratec USA creates world class Machinery and automated solutions for companies in all industries. We manufacture material handling systems, CNC Turning, fabrication and textile machinery, as well as other Integrated Technologies to improve productivity and process automation

The MWR120G is a high precision, multi process CNC Turning Machine with twin spindles and a Gantry. With a design based on automation at its core, Muratec's MWR120 series is a first-of-its-kind, front-facing, horizontal twin spindle CNC live tool chucker lathe equipped with Y-axis function. The total profit function of a company is;

$p(x) = -5x^2 + 130x + 37500$, where x is the production of the MWR120G machine.

Now naswer the following:

(i) What will be the production units when the profit is maximum? (1)

(ii) What will be the maximum profit? (1)

(iii) When the production is 2 units what will be the profit of the company? (2)

OR

(iii) What will be production of the company when the profit is $ 8700?

38. A man is watching an aeroplane which is at the coordinate point A(4, -1, 3) assuming that the man is at O (0, 0 , 0). At the same time he saw a bird at the coordinate points B(2, 0 ,4).

Based on the above information answer the following:

(i) Find the vector $\overrightarrow{AB}$. (1)

(ii) Fiind the distance between aeroplane and bird. (1)

(iii) Find the unit vector along vector $\overrightarrow{AB}$. (2)

OR

(iii) Find the direction cosine of vector $\overrightarrow{AB}$.